# Common Randomness Generation over Slow Fading Channels

Rami Ezzine,\* Moritz Wiese,\*<sup>‡</sup> Christian Deppe <sup>†</sup> and Holger Boche \*<sup>‡§</sup>

\*Technical University of Munich, Chair of Theoretical Information Technology, Munich, Germany

<sup>†</sup>Technical University of Munich, Institute for Communications Engineering, Munich, Germany

<sup>‡</sup>CASA – Cyber Security in the Age of Large-Scale Adversaries– Exzellenzcluster, Ruhr-Universität Bochum, Germany

<sup>§</sup>Munich Center for Quantum Science and Technology (MCQST), Schellingstr. 4, 80799 Munich, Germany

Email: {rami.ezzine, wiese, christian.deppe, boche}@tum.de

Abstract—This paper analyzes the problem of common randomness (CR) generation from correlated discrete sources aided by unidirectional communication over Single-Input Single-Output (SISO) slow fading channels with additive white Gaussian noise (AWGN) and arbitrary state distribution. Slow fading channels are practically relevant for wireless communications. We completely solve the SISO slow fading case by establishing its corresponding outage CR capacity using our characterization of its channel outage capacity. The generated CR could be exploited to improve the performance gain in the identification scheme. The latter is known to be more efficient than the classical transmission scheme in many new applications, which demand ultra-reliable low latency communication.

Index Terms—Common randomness, slow fading, outage capacity.

## I. INTRODUCTION

Common randomness (CR) of two terminals refers to a random variable observable to both, with low error probability. In many models, one terminal corresponds to the sender station and the other corresponds to the receiver station. The availability of this CR allows to implement correlated random protocols leading to developing potentially faster and more efficient algorithms [1]. CR generation plays a major role in sequential secret key generation [2]. In the context of secret key generation, CR is usually denoted by Information reconciliation. CR is also highly relevant in the identification scheme, an approach in communications developed by Ahlswede and Dueck [3] in 1989. In the identification framework, the encoder sends an identification message also called identity over the channel and the decoder is not interested in what the received message is. He wants to know if a specific message of special interest to him has been sent or not. The identification scheme is better suited than the classical transmission scheme for many new applications with high requirements on reliability and latency. These applications include several machine-to-machine and human-to-machine systems [4], the tactile internet [5], digital watermarking [6]–[8], industry 4.0 [9], molecular communications [10] [11], etc. Furthermore, [12] describes an interesting application where identification codes [13] can be used in autonomous driving. This is a typical use case for ultra-reliable low latency communication. Interestingly, it has been established that the resource CR can increase the identification capacity of channels [14]–[16].

Thus, by taking advantage of the resource CR, an enormous performance gain can be achieved in the identification task.

The problem of CR generation was initially introduced in [14], where unlike in the fundamental significant papers [17] [18], no secrecy requirements are imposed. In particular, the CR capacity of a model involving two correlated discrete sources with one-way communication over noiseless channels with limited capacity was established. The CR capacity is defined to be the maximum rate of CR generated by two terminals using the resources available in the model [14].

Recently, the results on CR capacity have been extended in [19] to SISO and point-to-point Multiple-Input Multiple-Output (MIMO) Gaussian channels, which are practically relevant in many communication situations including satellite and deep space communication links [20], wired and wireless communications, etc. The results on CR capacity over Gaussian channels have been used to establish a lower-bound on its corresponding correlation-assisted secure identification capacity in the log-log scale [19]. This lower bound can already exceed the secure identification capacity over Gaussian channels with randomized encoding elaborated in [21].

However, to the best of our knowledge, there are no results on the CR generation problem over fading channels. The generated CR can be exploited in the problem of correlationassisted identification over fading channels, which is, as far as we know, an open problem. The phenomenon of fading is one of the fundamental aspects in wireless communication. Fading refers to the deviation of a signal attenuation during wireless propagation with different variables such as time, rainfall, radio frequency. A common model for wireless communication is the fading channel model with additive white Gaussian noise (AWGN) [22]-[26]. In our work, the focus will be on SISO slow fading channels with AWGN and with arbitrary state distribution. In the slow fading scenario, the channel state is random but remains constant over the time-scale of transmission. The event of major interest here is outage. This arises when the channel state is so poor that no coding scheme is able to establish reliable communication at a certain target rate. We consider, as a capacity measure of the slow fading channel, the  $\eta$ -outage capacity defined to be the largest rate at which one can reliably communicate over the channel with probability greater or equal to  $1 - \eta$  [22] [23]. To the best of

our knowledge, no rigorous proof of the outage capacity for arbitrary state distribution is provided in the literature.

The main contribution of this paper consists in establishing first the  $\eta$ -outage capacity of SISO slow fading channels with AWGN and with arbitrary state distribution. Then, we extend the concept of outage to the CR generation problem over the slow fading channel by deriving a single-letter characterization of its corresponding  $\eta$ -outage CR capacity using our characterization of its  $\eta$ -outage capacity. In the CR generation framework, outage occurs when the channel state is so poor that the terminals cannot agree on a common random variable with high probability. The  $\eta$ -outage CR capacity is defined to be the maximum rate of CR generated by the terminals using the resources available in the model such that the outage probability does not exceed  $\eta$ .

*Paper outline:* The paper is organized as follows. In Section II, we present our system model, provide the key definitions and present the main results. The  $\eta$ -outage capacity is established in Section III. A rigorous proof of the  $\eta$ -outage CR capacity is provided in Section IV. The conclusion contains concluding remarks.

*Notation:*  $\mathbb{C}$  denotes the set of complex numbers and  $\mathbb{R}$  denotes the set of real numbers;  $H(\cdot)$  and  $h(\cdot)$  correspond to the entropy of discrete and continuous random variables, respectively;  $I(\cdot; \cdot)$  denotes the mutual information between two random variables;  $|\mathcal{K}|$  stands for the cardinality of the set  $\mathcal{K}$  and  $\mathcal{T}_U^n$  denotes the set of typical sequences of length n and of type  $P_U$ . For any random variables X, Y and Z, we use the operator  $X \Leftrightarrow Y \Leftrightarrow Z$  to indicate a Markov chain. Throughout the paper, log and exp are to the base 2.

#### **II. SYSTEM MODEL, DEFINITIONS AND MAIN RESULTS**

## A. System Model

Let a discrete memoryless multiple source  $P_{XY}$  with two components, with generic variables X and Y on alphabets  $\mathcal{X}$  and  $\mathcal{Y}$ , correspondingly, be given. The outputs of X are observed by Terminal A and those of Y by Terminal B. Both outputs have length n. Terminal A can send information to Terminal B over the following slow fading channel  $W_G$ :

$$z_i = Gt_i + \xi_i \quad i = 1 \dots n.$$

where  $t^n = (t_1, \ldots, t_n) \in \mathbb{C}^n$  and  $z^n = (z_1, \ldots, z_n) \in \mathbb{C}^n$ are channel input and output blocks, respectively. *G* models the complex gain, where we assume that both terminals *A* and *B* know the distribution of the gain *G* only.  $\xi^n = (\xi_1, \ldots, \xi_n) \in$  $\mathbb{C}^n$  models the noise sequence. We assume that the  $\xi_i s$  are i.i.d, where  $\xi_i \sim \mathcal{N}_{\mathbb{C}}(0, \sigma^2)$ ,  $i = 1 \dots n$ . We further assume that *G* and  $\xi^n$  are mutually independent and that  $(G, \xi^n)$  are independent of  $X^n, Y^n$ . There are no other resources available to both terminals.

A CR-generation protocol of block-length n consists of:

- 1) A function  $\Phi$  that maps  $X^n$  into a random variable K with alphabet  $\mathcal{K}$  generated by Terminal A.
- 2) A function f that maps  $X^n$  into some message  $\ell = f(X^n)$ .

3) A channel code  $\Gamma$  of length n for the channel  $W_G$ as defined in Definition 3, where each codeword  $t_{\ell} = (t_{\ell,1}, \ldots, t_{\ell,n})$  satisfies the following power constraint:

$$\frac{1}{n}\sum_{i=1}^{n} t_{\ell,i}^2 \le P.$$
 (1)

The random channel input sequence depending on  $X^n$  is denoted by  $T^n$ .

 A function Λ that maps Y<sup>n</sup> and the decoded message into a random variable L with alphabet K generated by Terminal B.

Such protocol induces a pair of random variable (K, L) that is called permissible, where it holds for some function  $\Psi$ , for Dbeing the channel decoder of  $\Gamma$  and for  $Z^n$  being the random channel output sequence that

$$K = \Phi(X^n), \quad L = \Psi(Y^n, Z^n) = \Lambda(Y^n, D(Z^n)).$$
(2)

This is illustrated in Fig. 1.



Fig. 1: Two-correlated source model with one-way communication over a SISO slow fading channel

## B. Rates and Capacities

We define first an achievable  $\eta$ -outage CR rate and the  $\eta$ outage CR capacity. This is an extension to the definition of an achievable CR rate and of the CR capacity introduced in [14].

**Definition 1.** Fix a non-negative constant  $\eta < 1$ . A number H is called an achievable  $\eta$ -outage common randomness rate if there exists a non-negative constant c such that for every  $\alpha > 0$  and  $\delta > 0$  and for sufficiently large n there exists a permissible pair of random variables (K, L) such that

$$\mathbb{P}\left[\mathbb{P}\left[K \neq L | G\right] \le \alpha\right] \ge 1 - \eta,\tag{3}$$

$$|\mathcal{K}| \le \exp(cn),\tag{4}$$

$$\frac{1}{n}H(K) > H - \delta. \tag{5}$$

**Remark 1.** Together with (3), the technical condition (4) ensures that for every  $\epsilon > 0$  and sufficiently large blocklength n the set

$$\mathcal{A} = \left\{ g \in \mathbb{C} : \left| \frac{H(K|G=g)}{n} - \frac{H(L|G=g)}{n} \right| < \epsilon \right\}$$

satisfies  $\mathbb{P}[\mathcal{A}] \geq 1 - \eta$ . This follows from the analogous statement in [14].

**Definition 2.** The  $\eta$ -outage common randomness capacity  $C_{\eta,CR}(P)$  is the maximum achievable  $\eta$ -outage common randomness rate.

Next, we define an achievable  $\eta$ -outage rate for the slow fading channel  $W_G$  and the corresponding  $\eta$ -outage capacity. For this purpose, we begin by providing the definition of a transmission-code for  $W_G$ .

**Definition 3.** A transmission-code  $\Gamma$  of length n and size  $|\Gamma|$  for the channel  $W_G$  is a family of pairs  $\{(t_{\ell}, \mathcal{D}_{\ell}), \ell = 1, ..., |\Gamma|\}$  such that for all  $\ell, j \in \{1, ..., |\Gamma|\}$ , we have:

$$\begin{aligned} \boldsymbol{t}_{\ell} \in \mathbb{C}^{n}, \quad \mathcal{D}_{\ell} \subset \mathbb{C}^{n}, \\ \frac{1}{n} \sum_{i=1}^{n} t_{\ell,i}^{2} \leq P \ \forall \ \boldsymbol{t}_{\ell} \in \mathbb{C}^{n}, \ \boldsymbol{t}_{\ell} = (t_{\ell,1}, \dots, t_{\ell,n}), \\ \mathcal{D}_{\ell} \cap \mathcal{D}_{j} = \varnothing, \quad \ell \neq j. \end{aligned}$$

Here,  $\mathbf{t}_{\ell}$ ,  $\ell = 1, \dots, |\Gamma|$  and  $\mathcal{D}_{\ell}$ ,  $\ell = 1, \dots, |\Gamma|$ , are the codewords and the decoding regions, respectively. The maximum error probability is a random variable depending on G and it is expressed as follows:

$$e(\Gamma, G) = \max_{\ell \in \{1, \dots, |\Gamma|\}} W_G(\mathcal{D}_{\ell}^c | \boldsymbol{t}_{\ell}).$$

**Remark 2.** Throughout the paper, we consider the maximum error probability criterion.

**Definition 4.** Let  $0 \le \eta < 1$ . A real number R is called an achievable  $\eta$ -outage rate of the channel  $W_G$  if for every  $\theta, \delta > 0$  there exists a code sequence  $(\Gamma_n)_{n=1}^{\infty}$  such that

$$\frac{\log|\Gamma_n|}{n} \ge R - \delta$$

and

$$\mathbb{P}[e(\Gamma_n, G) \le \theta] \ge 1 - \eta$$

for sufficiently large n.

**Definition 5.** The supremum of all achievable  $\eta$ -outage rates is called the  $\eta$ -outage capacity of the channel  $W_G$  and is denoted by  $C_{\eta}$ .

## C. Main Results

In this section, we propose a single-letter characterization of the  $\eta$ -outage channel capacity in Theorem 1 and of the  $\eta$ -outage CR capacity in Theorem 2. Theorem 1 and Theorem 2 are proved in Section III and Section IV, respectively.

**Theorem 1.** Let  $\gamma_0 = \sup\{\gamma : \mathbb{P}[|G| < \gamma] \le \eta\}$ . The  $\eta$ -outage capacity of the channel  $W_G$  is equal to

$$C_{\eta}(P) = \log\left(1 + \frac{P\gamma_0^2}{\sigma^2}\right)$$

**Theorem 2.** For the model described in Section II-A, the  $\eta$ -outage CR capacity is equal to

$$C_{\eta,CR}(P) = \max_{\substack{U \\ U \Leftrightarrow X \Leftrightarrow Y \\ I(U;X) - I(U;Y) \le C_{\eta}(P)}} I(U;X).$$

# III. PROOF OF THEOREM 1

Let

$$\gamma_0 = \sup\{\gamma : \mathbb{P}[|G| < \gamma] \le \eta\}$$

We will prove first the following lemma:

Lemma 1.

$$\mathbb{P}[|G| < \gamma_0] \le \eta_2$$

so the supremum actually is a maximum.

*Proof.* Let  $\gamma_n \nearrow \gamma_0$  be a sequence converging to  $\gamma_0$  from the left. Then

$$\{\gamma \in \mathbb{R} : \gamma < \gamma_0\} = \bigcup_{n=1}^{\infty} \{\gamma \in \mathbb{R} : \gamma < \gamma_n\}.$$

From the sigma-continuity of probability measures, it follows that

$$\mathbb{P}[|G| < \gamma_0] = \lim_n \mathbb{P}[|G| < \gamma_n] \le \eta.$$

Now, we begin with the direct proof of Theorem 1. We will show that

$$C_{\eta}(P) \ge \log\left(1 + \frac{P\gamma_0^2}{\sigma^2}\right). \tag{6}$$

Let  $\theta, \delta > 0$ . It is well-known that there exists a code sequence  $(\Gamma_n)_{n=1}^{\infty}$  and a blocklength  $n_0$  such that

$$\frac{\log|\Gamma_n|}{n} \ge \log\left(1 + \frac{P\gamma_0^2}{\sigma^2}\right) - \delta$$

and

$$e(\Gamma_n, \gamma_0) \le \theta$$

for  $n \geq n_0$ . The degradedness of the Gaussian channels implies that also

$$e(\Gamma_n, \gamma) \le \theta$$

for  $n \ge n_0$ , provided that  $\gamma \ge \gamma_0$ . Therefore for  $n \ge n_0$ , Lemma 1 implies

$$\mathbb{P}[e(\Gamma_n, G) \le \theta] \ge \mathbb{P}[|G| \ge \gamma_0] = 1 - \mathbb{P}[|G| < \gamma_0] \ge 1 - \eta.$$

This implies (6).

Next, we prove the converse of Theorem 1. We will show that

$$C_{\eta}(P) \le \log\left(1 + \frac{P\gamma_0^2}{\sigma^2}\right). \tag{7}$$

Suppose this were not true. Then there exists an  $\varepsilon > 0$  such that for all  $\theta, \delta > 0$  there exists a code sequence  $(\Gamma_n)_{n=1}^{\infty}$  satisfying

$$\frac{\log|\Gamma_n|}{n} \ge \log\left(1 + \frac{P(\gamma_0 + \varepsilon)^2}{\sigma^2}\right) - \delta \tag{8}$$

and

$$\mathbb{P}[e(\Gamma_n, G) \le \theta] \ge 1 - \eta \tag{9}$$

for sufficiently large *n*. The degradedness implies  $e(\Gamma_n, \gamma) \ge e(\Gamma_n, \gamma_0 + \varepsilon)$  for all  $\gamma \le \gamma_0 + \varepsilon$ . Since  $\delta$  may be arbitrary, we may choose it in such a way that the right-hand side of (8) is strictly larger than  $\log(1 + (P\gamma_0^2)/\sigma^2)$ . We define  $\gamma_1$  to be the solution of the equation

$$\log(1 + (P\gamma_1^2)/\sigma^2) = \log\left(1 + \frac{P(\gamma_0 + \varepsilon)^2}{\sigma^2}\right) - \delta.$$

 $\gamma_1$  is chosen such that the rate of the code sequence is greater than the capacity of the channel with gain G when  $|G| < \gamma_1$ . Therefore, it holds for large n that the error probability is greater than  $\theta$  when  $|G| < \gamma_1$ . It holds for large n that the error probability is greater than  $\theta$  when  $|G| < \gamma_1$ . It follows that

$$\mathbb{P}[e(\Gamma_n, G) > \theta] \ge \mathbb{P}[|G| < \gamma_1] > \eta$$

by the definition of  $\gamma_0$ , where we used that  $\gamma_1 > \gamma_0$  from the choice of  $\delta$ . This is a contradiction to (9), and so (7) must be true. This completes the proof of Theorem 1.

# IV. PROOF OF THEOREM 2

#### A. Converse Proof

Let (K, L) be a permissible pair according to the CRgeneration protocol introduced in Section II-A with power constraint as in (1). Let  $T^n$  and  $Z^n$  be the random channel input and output sequence, respectively. We further assume that (K, L) satisfies (3) (4) and (5). We are going to show for  $\alpha'(n) > 0$  that

$$\frac{H(K)}{n} \le \max_{\substack{U \in X \in Y\\I(U;X) - I(U;Y) \le C_{\eta}(P) + \alpha'(n)}} I(U;X),$$

where  $\lim_{n\to\infty} \alpha'(n)$  can be made arbitrarily small. In our proof, we will use the following lemma:

**Lemma 2.** (Lemma 17.12 in [27]) For arbitrary random variables S and R and sequences of random variables  $X^n$  and  $Y^n$ , it holds that

$$I(S, X^{n}|R) - I(S; Y^{n}|R)$$
  
=  $\sum_{i=1}^{n} I(S; X_{i}|X_{1} \dots X_{i-1}Y_{i+1} \dots Y_{n}R)$   
-  $\sum_{i=1}^{n} I(S; Y_{i}|X_{1} \dots X_{i-1}Y_{i+1} \dots Y_{n}R)$   
=  $n[I(S; X_{J}|V) - I(S; Y_{J}|V)],$ 

where  $V = X_1 \dots X_{J-1} Y_{J+1} \dots Y_n RJ$ , with J being a random variable independent of R, S,  $X^n$  and  $Y^n$  and uniformly distributed on  $\{1 \dots n\}$ .

Let J be a random variable uniformly distributed on  $\{1...n\}$  and independent of K,  $X^n$  and  $Y^n$ . We further

define  $U = KX_1 \dots X_{J-1}Y_{J+1} \dots Y_n J$ . Notice that

$$H(K) = I(K; X^{n})$$

$$\stackrel{(i)}{=} \sum_{i=1}^{n} I(K; X_{i} | X_{1} \dots X_{i-1})$$

$$= nI(K; X_{J} | X_{1} \dots X_{J-1}, J)$$

$$\stackrel{(ii)}{\leq} nI(U; X_{J}),$$

where (i) and (ii) follow from the chain rule for mutual information.

We will show next for  $\alpha'(n) > 0$  that

$$I(U;X_J) - I(U;Y_J) \le C_{\eta}(P) + \alpha'(n).$$

Applying Lemma 2 for S = K,  $R = \emptyset$  with  $V = X_1 \dots X_{J-1} Y_{J+1} \dots Y_n RJ$  yields

$$I(K; X^{n}) - I(K; Y^{n})$$
  
=  $n[I(K; X_{J}|V) - I(K; Y_{J}|V)]$   
 $\stackrel{(a)}{=} n[I(KV; X_{J}) - I(K; V) - I(KV; Y_{J}) + I(K; V)]$   
 $\stackrel{(b)}{=} n[I(U; X_{J}) - I(U; Y_{J})],$  (10)

where (a) follows from the chain rule for mutual information and (b) follows from U = KV. It results using (10) that

$$H(K|Y^{n}) = H(K) - I(K;Y^{n})$$

$$\stackrel{(c)}{=} H(K) - H(K|X^{n}) - I(K;Y^{n})$$

$$= I(K;X^{n}) - I(K;Y^{n})$$

$$= n[I(U;X_{J}) - I(U;Y_{J})], \qquad (11)$$

where (c) follows because  $K = \Phi(X^n)$  from (2).

Let  $\gamma_0 = \sup \{ \gamma : \mathbb{P}[|G| < \gamma] \le \eta \}$ . We consider for  $\epsilon > 0$  being arbitrarily small the set:

$$\Omega_1 \\ = \Big\{ g \in \mathbb{C} : \mathbb{P} \left[ K \neq L | G = g \right] \le \alpha \text{ and } |g| \le \gamma_0 + \epsilon \Big\}.$$

Lemma 3.

$$\mathbb{P}\left[\Omega_{1}\right] > 0.$$

*Proof.* From the definition of  $\gamma_0$ , we know that

 $\mathbb{P}\left[|G| < \gamma_0 + \epsilon\right] > \eta.$ 

This implies that

$$\mathbb{P}\left[|G| \le \gamma_0 + \epsilon\right] \ge \mathbb{P}\left[|G| < \gamma_0 + \epsilon\right]$$
  
>  $\eta.$  (12)

As a result

$$\mathbb{P}\left[|G| \le \gamma_0 + \epsilon\right] = \eta_1$$

where  $0 \le \eta < \eta_1 \le 1$ . It follows from (3) that

$$\begin{split} 1 &- \eta \\ &\leq \mathbb{P}\left[\mathbb{P}\left[K \neq L | G\right] \leq \alpha\right] \\ &= \mathbb{P}\left[|G| \leq \gamma_0 + \epsilon\right] \mathbb{P}\left[\mathbb{P}\left[K \neq L \mid G\right] \leq \alpha \mid |G| \leq \gamma_0 + \epsilon\right] \\ &+ \mathbb{P}\left[|G| > \gamma_0 + \epsilon\right] \mathbb{P}\left[\mathbb{P}\left[K \neq L \mid G\right] \leq \alpha \mid |G| > \gamma_0 + \epsilon\right] \\ &= \eta_1 \mathbb{P}\left[\mathbb{P}\left[K \neq L \mid G\right] \leq \alpha \mid |G| \leq \gamma_0 + \epsilon\right] \\ &+ (1 - \eta_1) \mathbb{P}\left[\mathbb{P}\left[K \neq L \mid G\right] \leq \alpha \mid |G| > \gamma_0 + \epsilon\right] \\ &\leq \mathbb{P}\left[\mathbb{P}\left[K \neq L \mid G\right] \leq \alpha \mid |G| \leq \gamma_0 + \epsilon\right] + (1 - \eta_1) \\ &< \mathbb{P}\left[\mathbb{P}\left[K \neq L \mid G\right] \leq \alpha \mid |G| \leq \gamma_0 + \epsilon\right] + (1 - \eta), \end{split}$$

where we used that  $1 - \eta_1 < 1 - \eta$ . This means that

$$\mathbb{P}\left[\mathbb{P}\left[K \neq L \mid G\right] \le \alpha \mid |G| \le \gamma_0 + \epsilon\right] > 0.$$

In addition, since  $\eta_1 > 0$ , it follows that

$$\mathbb{P}\left[\mathbb{P}\left[K \neq L \mid G\right] \leq \alpha, |G| \leq \gamma_0 + \epsilon\right] > 0.$$

It follows that

$$\mathbb{P}\left[\Omega_{1}\right] > 0.$$

Next, we define  $\tilde{G}$  to be a random variable, independent of  $X^n, Y^n$  and  $\xi^n$ , with alphabet  $\Omega_1$  such that for every Borel set  $\mathcal{A} \subseteq \mathbb{C}$ , it holds that

$$\mathbb{P}\left[\tilde{G} \in \mathcal{A}\right] = \mathbb{P}\left[G \in \mathcal{A} | G \in \Omega_1\right].$$

We fix the CR generation protocol and change the state distribution of the slow fading channel. We obtain the following new channel:

$$\tilde{Z}_i = \tilde{G}T_i + \xi_i \quad i = 1 \dots n,$$

where  $\tilde{Z}^n$  is the new output sequence. We further define  $\tilde{L}$  such that

$$\tilde{L} = \Psi(Y^n, \tilde{Z}^n).$$

Clearly, it holds that

$$\mathbb{P}\left[K \neq \tilde{L} | \tilde{G} = g\right] \le \alpha \quad \forall g \in \Omega_1,$$
(13)

and that

$$\log(1 + \frac{|g|^2 P}{\sigma^2}) \le \log(1 + \frac{(\gamma_0 + \epsilon)^2 P}{\sigma^2}) \quad \forall g \in \Omega_1.$$
 (14)

Furthermore, since  $\xi_i \sim \mathcal{N}_{\mathbb{C}}(0, \sigma^2), i = 1 \dots n$ , it follows from (1) that for  $i = 1 \dots n$ ,

$$I(T_i, \tilde{Z}_i | \tilde{G} = g) \le \log(1 + \frac{|g|^2 P}{\sigma^2}) \quad \forall \ g \in \Omega_1.$$
 (15)

We have:

$$\begin{split} H(K|Y^n) &= H(K|\tilde{G},Y^n) \\ &= H(K|\tilde{G},Y^n,\tilde{Z}^n) + I(K;\tilde{Z}^n|\tilde{G},Y^n), \end{split}$$

where we used that  $\tilde{G}$  is independent of  $(K, Y^n)$ . On the one hand, we have:

$$H\left(K|\tilde{Z}^{n},\tilde{G},Y^{n}\right) \stackrel{(a)}{\leq} H\left(K|\tilde{L},\tilde{G}\right)$$

$$\stackrel{(b)}{\leq} \mathbb{E}\left[1 + \log|\mathcal{K}|\mathbb{P}[K \neq \tilde{L}|\tilde{G}]\right]$$

$$= 1 + \log|\mathcal{K}|\mathbb{E}\left[P[K \neq \tilde{L}|\tilde{G}]\right]$$

$$\stackrel{(c)}{\leq} 1 + \alpha \log|\mathcal{K}|$$

$$\stackrel{(d)}{\leq} 1 + \alpha \ cn,$$

where (a) follows from  $\tilde{L} = \Psi(Y^n, \tilde{Z}^n)$ , (b) follows from Fano's Inequality, (c) follows from (13) and (d) follows from  $\log |\mathcal{K}| \leq cn$  in (4).

On the other hand, we have:

$$\begin{split} I(K;\tilde{Z}^{n}|\tilde{G},Y^{n}) &\leq I(X^{n}K;\tilde{Z}^{n}|\tilde{G},Y^{n}) \\ &\stackrel{(a)}{\leq} I(T^{n};\tilde{Z}^{n}|\tilde{G},Y^{n}) \\ &= h(\tilde{Z}^{n}|\tilde{G},Y^{n}) - h(\tilde{Z}^{n}|T^{n},\tilde{G},Y^{n}) \\ &\stackrel{(b)}{=} h(\tilde{Z}^{n}|\tilde{G},Y^{n}) - h(\tilde{Z}^{n}|\tilde{G},T^{n}) \\ &\stackrel{(c)}{\leq} h(\tilde{Z}^{n}|\tilde{G}) - h(\tilde{Z}^{n}|\tilde{G},T^{n}) \\ &= I(T^{n};\tilde{Z}^{n}|\tilde{G}) \\ &\stackrel{(d)}{=} \sum_{i=1}^{n} I(\tilde{Z}_{i};T^{n}|\tilde{G},\tilde{Z}^{i-1}) \\ &= \sum_{i=1}^{n} h(\tilde{Z}_{i}|\tilde{G},\tilde{Z}^{i-1}) - h(\tilde{Z}_{i}|\tilde{G},T^{n},\tilde{Z}^{i-1}) \\ &\stackrel{(e)}{=} \sum_{i=1}^{n} h(\tilde{Z}_{i}|\tilde{G},\tilde{Z}^{i-1}) - h(\tilde{Z}_{i}|\tilde{G},T_{i}) \\ &\stackrel{(f)}{\leq} \sum_{i=1}^{n} h(\tilde{Z}_{i}|\tilde{G}) - h(\tilde{Z}_{i}|\tilde{G},T_{i}) \\ &= \sum_{i=1}^{n} I(T_{i};\tilde{Z}_{i}|\tilde{G}) \\ &\stackrel{(g)}{\leq} n\mathbb{E} \left[ \log(1 + \frac{|\tilde{G}|^{2}P}{\sigma^{2}}) \right] \\ &\stackrel{(h)}{\leq} n\log(1 + \frac{(\gamma_{0} + \epsilon)^{2}P}{\sigma^{2}}) \\ &\stackrel{(i)}{=} n(C_{n}(P) + \epsilon'), \end{split}$$

with  $\epsilon'$  being arbitrarily small, where (a) follows from the Data Processing Inequality because  $Y^n \Leftrightarrow X^n K \Leftrightarrow \tilde{G}T^n \Leftrightarrow \tilde{Z}^n$ forms a Markov chain, (b) follows because  $Y^n \Leftrightarrow X^n K \Leftrightarrow \tilde{G}T^n \Leftrightarrow \tilde{Z}^n$  forms a Markov chain, (c)(f) follow because conditioning does not increase entropy, (d) follows from the chain rule for mutual information, (e) follows because  $T_1 \dots T_{i-1}T_{i+1} \dots T_n \tilde{Z}^{i-1} \Leftrightarrow \tilde{G}T_i \Leftrightarrow \tilde{Z}_i$  forms a Markov chain, (q) follows from (15) and (h) follows from (14) and (i) follows from Theorem 1 using that  $\epsilon$  is arbitrarily small. This proves that for  $0 \le \eta < 1$ 

$$\frac{H(K|Y^n)}{n} \le C_\eta(P) + \alpha'(n), \tag{16}$$

where  $\alpha'(n) = \frac{1}{n} + \alpha c + \epsilon' > 0.$ 

From (11) and (16), we deduce that for  $0 \leq \eta < 1$ 

$$I(U;X_J) - I(U;Y_J) \le C_{\eta}(P) + \alpha'(n),$$

where  $U \Leftrightarrow X_J \Leftrightarrow Y_J$ .

Since the joint distribution of  $X_J$  and  $Y_J$  is equal to  $P_{XY}$ ,  $\frac{H(K)}{n}$  is upper-bounded by I(U;X) subject to  $I(U;X) - I(U;Y) \le C_{\eta}(P) + \alpha'(n)$  with U satisfying  $U \Leftrightarrow X \Leftrightarrow Y$ . As a result, for  $\alpha'(n) > 0$ , it holds that

$$\frac{H(K)}{n} \le \max_{\substack{U \\ U \notin X \notin Y \\ I(U;X) - I(U;Y) \le C_{\eta}(P) + \alpha'(n)}} I(U;X).$$

Here,  $\lim_{n\to\infty} \alpha'(n)$  can be made arbitrarily small. This completes the converse proof.

# B. Direct Proof

We extend the coding scheme provided in [14] to slow fading channels. By continuity, it suffices to show that

$$C'_{\eta,CR}(P) = \max_{\substack{U \\ U \Leftrightarrow X \Leftrightarrow Y \\ I(U;X) - I(U;Y) \le C'}} I(U;X)$$

is an achievable  $\eta$ -outage CR rate for every  $C' < C_{\eta}(P)$ . Let U be a random variable satisfying  $I(U; X) - I(U; Y) \leq C'$ . We are going to show that H = I(U; X) is an achievable  $\eta$ -outage CR rate. Without loss of generality, assume that the distribution of U is a possible type for block length n. Let

$$N_1 = \exp(n[I(U;X) - I(U;Y) + 3\delta])$$
  

$$N_2 = \exp(n[I(U;Y) - 2\delta]).$$

)

For each pair (i, j) with  $1 \le i \le N_1$  and  $1 \le j \le N_2$ , we define a random sequence  $U_{i,j} \in U^n$  of type  $P_U$ . Each realization  $u_{i,j}$  of  $U_{i,j}$  is known to both terminals. This means that  $N_1$  codebooks  $C_i, 1 \le i \le N_1$ , are known to both terminals, where each codebook contains  $N_2$  sequences  $u_{i,j}, j = 1 \dots N_2$ .

It holds for every X-typical  $\boldsymbol{x}$  that

$$\mathbb{P}[\exists (i,j) \text{ s.t } \boldsymbol{U}_{ij} \in \mathcal{T}_{U|X}^n(\boldsymbol{x}) | X^n = \boldsymbol{x}] \ge 1 - \exp(-\exp(nc')),$$

for a suitable c' > 0, as in the proof of Theorem 4.1 of [14]. For K(x), we choose a sequence  $u_{ij}$  jointly typical with x(either one if there are several). Let f(x) = i if  $K(x) \in C_i$ . If no such  $u_{i,j}$  exists, then  $f(x) = N_1 + 1$  and K(x) is set to a constant sequence  $u_0$  different from all the  $u_{ijs}$  and known to both terminals. Since  $C' < C_{\eta}(P)$ , we choose  $\delta$  to be sufficiently small such that

$$\frac{\log \|f\|}{n} = \frac{\log(N_1 + 1)}{n}$$
$$\leq C_{\eta}(P) - \delta', \tag{17}$$

for some  $\delta' > 0$ , where ||f|| refers to the cardinality of the set of messages  $\{i^* = f(x)\}$ . This is the same notation used in [27]. The message  $i^* = f(x)$ , with  $i^* \in \{1, \ldots, N_1 + 1\}$ , is encoded to a sequence t using a code sequence  $(\Gamma_n^*)_{n=1}^{\infty}$  with rate  $\frac{\log|\Gamma_n^*|}{n} = \frac{\log||f||}{n}$  satisfying (17) and with error probability  $e(\Gamma_n^*, G)$  satisfying:

$$\mathbb{P}\left[e(\Gamma_n^{\star}, G) \le \theta\right] \ge 1 - \eta, \tag{18}$$

where  $\theta$  is sufficiently small for sufficiently large n. From the definition of the  $\eta$ -outage capacity, we know that such a code sequence exists. The sequence t is sent over the slow fading channel. Let z be the channel output sequence. Terminal B decodes the message  $\tilde{i}^*$  from the knowledge of z. Let  $L(y, \tilde{i}^*) = u_{\tilde{i}^*, j}$  if  $u_{\tilde{i}^*, j}$  and y are UY-typical. If there is no such  $u_{\tilde{i}^*, j}$  or there are several, L is set equal to  $u_0$  (since K and L must have the same alphabet). Now, we are going to show that the requirements in (3) (4) and (5) are satisfied. Clearly, (4) is satisfied for c = 2(H(X) + 1), n sufficiently large:

$$\begin{aligned} |\mathcal{K}| &= N_1 N_2 + 1 \\ &= \exp(n \left[ I(U; X) + \delta \right]) + 1 \\ &\leq \exp(2n \left[ H(X) + 1 \right]). \end{aligned}$$

We define next for a fixed  $u_{i,j}$  the set

$$\Omega = \{ \boldsymbol{x} \in \mathcal{X}^n \text{ s.t. } (\boldsymbol{x}, \boldsymbol{u}_{i,j}) \text{ jointly typical} \}.$$

As shown in [14], it holds that

$$\begin{split} \mathbb{P}[K = \boldsymbol{u}_{i,j}] \\ &= \sum_{\boldsymbol{x} \in \Omega} \mathbb{P}[K = \boldsymbol{u}_{i,j} | X^n = \boldsymbol{x}] P_X^n(\boldsymbol{x}) \\ &+ \sum_{\boldsymbol{x} \in \Omega^c} \mathbb{P}[K = \boldsymbol{u}_{i,j} | X^n = \boldsymbol{x}] P_X^n(\boldsymbol{x}) \\ \stackrel{(i)}{=} \sum_{\boldsymbol{x} \in \Omega} \mathbb{P}[K = \boldsymbol{u}_{i,j} | X^n = \boldsymbol{x}] P_X^n(\boldsymbol{x}) \\ &\leq \sum_{\boldsymbol{x} \in \Omega} P_X^n(\boldsymbol{x}) \\ &= P_X^n(\{\boldsymbol{x} : (\boldsymbol{x}, \boldsymbol{u}_{i,j}) \text{ jointly typical}\}) \\ &= \exp\left(-nI(U; X) + o(n)\right), \end{split}$$

where (i) follows because for  $(\mathbf{x}, \boldsymbol{u}_{i,j})$  being not jointly typical, we have  $\mathbb{P}[K = \boldsymbol{u}_{i,j} | X^n = \boldsymbol{x}] = 0$ . This yields

$$H(K) \ge nI(U; X) + o(n)$$
  
=  $nH + o(n)$ .

Thus, (5) is satisfied. Now, it remains to prove that (3) is satisfied. Let  $M = U_{11} \dots U_{N_1N_2}$ . We define the following two sets which depend on M:

$$S_1(\boldsymbol{M}) = \{(\boldsymbol{x}, \boldsymbol{y}) : (K(\boldsymbol{x}), \boldsymbol{x}, \boldsymbol{y}) \in \mathcal{T}_{UXY}^n\}$$

and

$$S_2(\boldsymbol{M}) = \{(\boldsymbol{x}, \boldsymbol{y}) : (\boldsymbol{x}, \boldsymbol{y}) \in S_1(\boldsymbol{M}) \text{ s.t. } \exists \ \boldsymbol{U}_{i\ell} \neq \boldsymbol{U}_{ij} = K(\boldsymbol{x}) \\ \text{jointly typical with } \boldsymbol{y} \text{ (with the same first index } i) \}.$$

It is proved in [14] that

$$\mathbb{E}_{\boldsymbol{M}}\left[P_{XY}^{n}(S_{1}^{c}(\boldsymbol{M}))+P_{XY}^{n}(S_{2}(\boldsymbol{M}))\right]\leq\beta,\qquad(19)$$

where  $\beta$  is exponentially small for sufficiently large n.

**Remark 3.**  $P_{XY}^n(S_1^c(M))$  and  $P_{XY}^n(S_2(M))$  are here random variables depending on M.

We choose a realization  $m = u_{11} \dots u_{N_1N_2}$  satisfying:

$$P_{XY}^n(S_1^c(\boldsymbol{m})) + P_{XY}^n(S_2(\boldsymbol{m})) \le \beta.$$

From (19), we know that such a realization exists. Now, we define the following event:

$$\mathcal{D}_{\boldsymbol{m}} = "K(X^n)$$
 is equal to none of the  $\boldsymbol{u}_{i,j}s$ ".

We further define  $I^* = f(X^n)$  to be the random variable modeling the message encoded by Terminal A and  $\tilde{I}^*$  to be the random variable modeling the message decoded by Terminal B. We have:

$$\mathbb{P}[K \neq L|G] = \mathbb{P}[K \neq L|G, I^{\star} = \tilde{I}^{\star}]\mathbb{P}[I^{\star} = \tilde{I}^{\star}|G] \\ + \mathbb{P}[K \neq L|G, I^{\star} \neq \tilde{I}^{\star}]\mathbb{P}[I^{\star} \neq \tilde{I}^{\star}|G] \\ \leq \mathbb{P}[K \neq L|G, I^{\star} = \tilde{I}^{\star}] + \mathbb{P}[I^{\star} \neq \tilde{I}^{\star}|G].$$

Here:

$$\begin{split} \mathbb{P}[K \neq L|G, I^{\star} = I^{\star}] \\ &= \mathbb{P}[K \neq L|G, I^{\star} = \tilde{I}^{\star}, \mathcal{D}_{\boldsymbol{m}}] \mathbb{P}[\mathcal{D}_{\boldsymbol{m}}|G, I^{\star} = \tilde{I}^{\star}] \\ &+ \mathbb{P}[K \neq L|G, I^{\star} = \tilde{I}^{\star}, \mathcal{D}_{\boldsymbol{m}}^{c}] \mathbb{P}[\mathcal{D}_{\boldsymbol{m}}^{c}|G, I^{\star} = \tilde{I}^{\star}] \\ \stackrel{(i)}{=} \mathbb{P}[K \neq L|G, I^{\star} = \tilde{I}^{\star}, \mathcal{D}_{\boldsymbol{m}}^{c}] \mathbb{P}[\mathcal{D}_{\boldsymbol{m}}^{c}|G, I^{\star} = \tilde{I}^{\star}] \\ &\leq \mathbb{P}[K \neq L|G, I^{\star} = \tilde{I}^{\star}, \mathcal{D}_{\boldsymbol{m}}^{c}], \end{split}$$

where (i) follows from  $\mathbb{P}[K \neq L | G, I^* = \tilde{I}^*, \mathcal{D}_m] = 0$ , since conditioned on  $G, I^* = \tilde{I}^*$  and  $\mathcal{D}_m$ , we know that K and L are both equal to  $u_0$ . It follows that

$$\mathbb{P}[K \neq L|G]$$

$$\leq \mathbb{P}[K \neq L|G, I^{\star} = \tilde{I}^{\star}, \mathcal{D}_{\boldsymbol{m}}^{c}] + \mathbb{P}[I^{\star} \neq \tilde{I}^{\star}|G]$$

$$\leq P_{XY}^{n} \left(S_{1}^{c}(\boldsymbol{m}) \cup S_{2}(\boldsymbol{m})\right) + \mathbb{P}[I^{\star} \neq \tilde{I}^{\star}|G]$$

$$\stackrel{(a)}{\leq} P_{XY}^{n} \left(S_{1}^{c}(\boldsymbol{m})\right) + P_{XY}^{n} \left(S_{2}(\boldsymbol{m})\right) + \mathbb{P}[I^{\star} \neq \tilde{I}^{\star}|G]$$

$$\leq \beta + \mathbb{P}[I^{\star} \neq \tilde{I}^{\star}|G],$$

where (a) follows from the union bound. From (18), we know that

$$\mathbb{P}\left[\mathbb{P}\left[I^{\star} \neq \tilde{I}^{\star} | G\right] \leq \theta\right] \geq 1 - \eta.$$

We have:

$$\mathbb{P}\left[I^{\star} \neq \tilde{I}^{\star}|G\right] \leq \theta \implies \mathbb{P}[K \neq L|G] \leq \beta + \theta.$$

By choosing  $\alpha = \beta + \theta$ , we have:

$$\mathbb{P}\left[I^{\star} \neq \tilde{I}^{\star}|G\right] \leq \theta \implies \mathbb{P}[K \neq L|G] \leq \alpha.$$

Thus:

$$\mathbb{P}\left[\mathbb{P}[K \neq L | G] \le \alpha\right] \ge \mathbb{P}\left[\mathbb{P}\left[I^{\star} \neq \tilde{I}^{\star} | G\right] \le \theta\right]$$
$$\ge 1 - \eta.$$

Here,  $\alpha$  is arbitrarily small for sufficiently large *n*. This completes the direct proof.

# V. CONCLUSION

In this paper, we have examined the problem of common randomness generation over slow fading channels for their practical relevance in many situations in wireless communications. The generated CR can be exploited in the identification scheme to improve the performance gain. We established a single-letter characterization of the outage CR capacity over slow fading channels with AWGN and with arbitrary state distribution using our characterization of its corresponding channel outage capacity. As a future work, it would be interesting to study the problem of CR generation over singleuser MIMO slow fading channels since it is known that, compared to SISO systems, point-to-point MIMO communication systems offer higher rates, more reliability and resistance to interference. Future research might also focus on studying the problem of CR generation over fast fading channels.

## **ACKNOWLEDGMENTS**

We thank the German Research Foundation (DFG) within the Gottfried Wilhelm Leibniz Prize under Grant BO 1734/20-1 for their support of H. Boche and M. Wiese. Thanks also go to the German Federal Ministry of Education and Research (BMBF) within the national initiative for "Post Shannon Communication (NewCom)" with the project "Basics, simulation and demonstration for new communication models" under Grant 16KIS1003K for their support of H. Boche, R. Ezzine and with the project "Coding theory and coding methods for new communication models" under Grant 16KIS1005 for their support of C. Deppe. Further, we thank the German Research Foundation (DFG) within Germany's Excellence Strategy EXC-2111—390814868 and EXC-2092 CASA - 390781972 for their support of H. Boche and M. Wiese.

#### REFERENCES

- M. Sudan, H. Tyagi, and S. Watanabe, "Communication for generating correlation: A unifying survey," *IEEE Transactions on Information Theory*, vol. 66, no. 1, pp. 5–37, 2020.
- [2] M. Bloch and J. Barros, *Physical-Layer Security: From Information Theory to Security Engineering*. Cambridge University Press, 2011.
- [3] R. Ahlswede and G. Dueck, "Identification via channels," *IEEE Transactions on Information Theory*, vol. 35, no. 1, pp. 15–29, 1989.
- [4] H. Boche and C. Deppe, "Secure identification for wiretap channels; robustness, super-additivity and continuity," *IEEE Transactions on Information Forensics and Security*, vol. 13, no. 7, pp. 1641–1655, 2018.
- [5] G. P. Fettweis, "The tactile internet: Applications and challenges," *IEEE Vehicular Technology Magazine*, vol. 9, no. 1, pp. 64–70, 2014.
- [6] P. Moulin, "The role of information theory in watermarking and its application to image watermarking," *Signal Processing*, vol. 81, no. 6, pp. 1121 – 1139, 2001, special section on Information theoretic aspects of digital watermarking. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0165168401000378

- [7] R. Ahlswede and N. Cai, Watermarking Identification Codes with Related Topics on Common Randomness. Berlin, Heidelberg: Springer Berlin Heidelberg, 2006, pp. 107–153.
- [8] Y. Steinberg and N. Merhav, "Identification in the presence of side information with application to watermarking," *IEEE Transactions on Information Theory*, vol. 47, no. 4, pp. 1410–1422, 2001.
- [9] Y. Lu, "Industry 4.0: A survey on technologies, applications and open research issues," *Journal of Industrial Information Integration*, vol. 6, pp. 1 – 10, 2017.
- [10] S. F. Bush, J. L. Paluh, G. Piro, V. Rao, R. V. Prasad, and A. Eckford, "Defining communication at the bottom," *IEEE Transactions on Molecular, Biological and Multi-Scale Communications*, vol. 1, no. 1, pp. 90–96, 2015.
- [11] W. Haselmayr, A. Springer, G. Fischer, C. Alexiou, H. Boche, P. Hoeher, F. Dressler, and R. Schober, "Integration of molecular communications into future generation wireless networks," 2019.
- [12] H. Boche and C. Arendt, "Communication method, mobile unit, interface unit, and communication system," 2021, patent number: 10959088.
- [13] S. Derebeyoğlu, C. Deppe, and R. Ferrara, "Performance analysis of identification codes," *Entropy*, vol. 22, no. 10, p. 1067, 2020.
- [14] R. Ahlswede and I. Csiszar, "Common randomness in information theory and cryptography. II. CR capacity," *IEEE Transactions on Information Theory*, vol. 44, no. 1, pp. 225–240, 1998.
- [15] R. Ahlswede, "General theory of information transfer: Updated," *Discrete Applied Mathematics*, vol. 156, pp. 1348–1388, 05 2008.
- [16] A. Ahlswede, I. Althöfer, C. Deppe, and T. Ulrich, *Identification and Other Probabilistic Models Rudolf Ahlswede's Lectures on Information Theory 6*, 1st ed. Springer-Verlag, 2021, vol. 16.
- [17] R. Ahlswede and I. Csiszar, "Common randomness in information theory and cryptography. I. secret sharing," *IEEE Transactions on Information Theory*, vol. 39, no. 4, pp. 1121–1132, 1993.
- [18] U. M. Maurer, "Secret key agreement by public discussion from common information," *IEEE Transactions on Information Theory*, vol. 39, no. 3, pp. 733–742, 1993.
- [19] R. Ezzine, W. Labidi, H. Boche, and C. Deppe, "Common randomness generation and identification over gaussian channels," in *GLOBECOM* 2020 - 2020 IEEE Global Communications Conference (GLOBECOM), 2020, pp. 1–6.
- [20] D. D. N. Bevan, V. T. Ermolayev, A. G. Flaksman, I. M. Averin, and P. M. Grant, "Gaussian channel model for macrocellular mobile propagation," in 2005 13th European Signal Processing Conference, 2005, pp. 1–4.
- [21] W. Labidi, C. Deppe, and H. Boche, "Secure identification for Gaussian channels," in *ICASSP 2020 - 2020 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, 2020, pp. 2872– 2876.
- [22] A. Goldsmith, Wireless Communications. Cambridge University Press, 2005.
- [23] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*. New York, NY, USA: Cambridge University Press, 2005.
- [24] E. Biglieri, J. Proakis, and S. Shamai, "Fading channels: informationtheoretic and communications aspects," *IEEE Transactions on Information Theory*, vol. 44, no. 6, pp. 2619–2692, 1998.
- [25] L. H. Ozarow, S. Shamai, and A. D. Wyner, "Information theoretic considerations for cellular mobile radio," *IEEE Transactions on Vehicular Technology*, vol. 43, no. 2, pp. 359–378, 1994.
- [26] X. Yang, "Capacity of fading channels without channel side information," *CoRR*, vol. abs/1903.12360, 2019. [Online]. Available: http://arxiv.org/abs/1903.12360
- [27] I. Csiszár and J. Körner, Information Theory: Coding Theorems for Discrete Memoryless Systems, 2nd ed. Cambridge University Press, 2011.