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Efficient Calculation of Adversarial Examples for Bayesian Neural Networks

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Abstract

Calculating adversarial examples for Bayesian neural networks is cumbersome. Due to the inherent stochasticity, the gradient of the network can only be reliable approximated by sampling multiple times from the posterior, leading to a greatly increased computational cost. In this paper we propose to efficiently attack Bayesian neural networks with adversarial examples calculated for a deterministic network with parameters given by the mean of the posterior distribution. We show in a series of experiments, that the proposed approach can be used to effectively attack Bayesian neural networks while using 4.2 times less of the resources of existing adversarial example estimation methods with comparable strength. We demonstrate that this is especially helpful during adversarial training, when multiple different model configuration need to be evaluated.

1. Introduction

Machine learning is ubiquitous. Starting from breakthroughs in computer vision (Krizhevsky et al., 2012; He et al., 2016) and natural language processing (Collobert and Weston, 2008); machine learning systems have evolved to play games at human level (Silver et al., 2016; Vinyals et al., 2019; Mnih et al., 2015), leverage drug discovery (Senior et al., 2020), recognize speech at the level of human listeners (Ram et al., 2017), and drive cars (Shankland, 2019). Given the influence on more and more parts of human live, the existence of adversarial examples (AEs) (Biggio et al., 2014; Szegedy et al., 2014) is worrisome. These small perturbations, added to benign inputs, cause even state-of-the-art models to misbehave.

Recent work suggest Bayesian neural networks (BNNs) (Neal, 1995) as a more robust alternative (Feinman et al., 2017; Gal and Smith, 2018; Liu et al., 2019; Bekasov and Murray, 2018). Carbone et al. (2020) even show that under certain assumptions the gradient has zero expectation under the posterior distribution and hence any gradient based attack in the limit is ineffective. Despite these theoretical efforts BNNs have successfully been attacked by taking multiple samples from the posterior distribution to approximate their gradient as proposed as a general attack strategy for networks with stochastic gradients (Athalye et al., 2018; Zimmermann, 2019). While effective, this approach is computationally costly. This is less of a problem for a malicious actor, since they simply need to find one AE to serve their purpose. Yet, from the defending parties point of view, evaluating several different models with regards to adversarial robustness can quickly become a costly endeavor. This

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cost is severely multiplied in the context of adversarial training (Madry et al., 2017), where adversarial examples are used in the training process to generate robust models.

Recognizing these problems, we introduce `MEAN`—an efficient way to compute AEs for BNNs, which can also be adopted to other types of stochastic networks. Our approach employs the means of the posterior distribution as the parameters of a deterministic network of equal architecture. We then employ this network to estimate AEs which are then used to attack the original BNN. In a first experiment, we demonstrate that `MEAN` can be used to generate effective AEs while using substantially fewer resources than existing approaches. In a second experiment, we evaluate `MEAN` in the setting of adversarial training, where a computational efficient method allows a defending party to evaluate multiple different models with a much smaller computational budget.

2. Background and Approach

Notation: We denote with $\mathcal{D} = \{(\boldsymbol{x}_i, \boldsymbol{y}_i)_{i=1}^n\}$ the training data where $\boldsymbol{x}_i$ are inputs, $\boldsymbol{y}_i$ the corresponding outputs and $\boldsymbol{\theta}$ the network's parameters.

2.1. Background

Bayesian neural networks serve as a mathematical quantified way to capture model uncertainty by marginalizing over the posterior distribution of the network's parameters:

$$p(\boldsymbol{y}|\boldsymbol{x}, \mathcal{D}) = \int p(\boldsymbol{y}|\boldsymbol{x}, \boldsymbol{\theta}) p(\boldsymbol{\theta}|\mathcal{D}) d\boldsymbol{\theta} \approx \frac{1}{S} \sum_{s=1}^{S} p(\boldsymbol{y}|\boldsymbol{x}, \boldsymbol{\theta}_s) \ , \tag{1}$$

where the last approximation is the Monte Carlo approximation of this intractable integral. Central to the training of BNNs is the derivation of the posterior distribution $p(\boldsymbol{\theta}|\mathcal{D})$. In this work we use variational inference (Graves, 2011; Kingma et al., 2015; Louizos and Welling, 2016) where the true posterior is approximated via a simpler distribution $q(\boldsymbol{\theta})$ by maximizing the so called *evidence lower bound* (ELBO):

$$ELBO = \sum_{i=1}^{n} \mathbb{E}_{q(\boldsymbol{\theta})}[p(\boldsymbol{y}_i|\boldsymbol{x}_i, \boldsymbol{\theta})] - KL[q(\boldsymbol{\theta})||p(\boldsymbol{\theta})] \ , \tag{2}$$

where the last term is the Kullback-Leibler divergence between the approximate posterior and some prior distribution.

Adversarial examples are manipulated inputs aiming at changing the networks' behavior. They are created by adding a well-calibrated perturbation δ to a benign input. One of the simplest methods to determine this δ is the *Fast Gradient Sign Method* (FGSM) (Goodfellow et al., 2015), in which the AE is created by adding an ϵ-step into the direction of the gradient sign of some loss function L, e.g.:

$$\boldsymbol{x}_{adv} = \boldsymbol{x} + \epsilon \cdot sign\left(\nabla_{\boldsymbol{x}} L\left(\boldsymbol{\theta}, \boldsymbol{x}, \boldsymbol{y}\right)\right) \ . \tag{3}$$

In practice the δ is usually limited by an L_p-norm to avoid changing the benign input too much. Improving FGSM, *Projected Gradient Decent* (PGD) (Madry et al., 2017) is an iterative attack which iterativly applies FGSM with a smaller stepsize than ϵ.

To calculate AE for BNNs, multiple draws from the posterior are needed for reliably approximating the stochastic gradient. For the FGSM attack this results in:

$$\boldsymbol{x}_{adv,BNN} = \boldsymbol{x} + \epsilon \cdot sign\left(\nabla_{\boldsymbol{x}} L\left(\boldsymbol{\theta}_{t=1}^{T}, \boldsymbol{x}, \boldsymbol{y}\right)\right) \quad . \tag{4}$$

In this work, we use the loss function introduced by Carlini and Wagner (2017) instead of the more commonly used cross-entropy loss. The loss function is given by

$$L_{\text{CW}}(\boldsymbol{x}, \boldsymbol{\theta}) = \max(\max_{i \neq t}(Z(\boldsymbol{x}, \boldsymbol{\theta})_i) - Z(\boldsymbol{x}, \boldsymbol{\theta})_t, \mathbf{0}) \quad , \tag{5}$$

where $Z(\boldsymbol{x}, \boldsymbol{\theta})_t$ is an arbitrary logit of an output node for input $\boldsymbol{x}$ and parameters $\boldsymbol{\theta}$. We noticed that our networks often saturated the softmax function, i.e., the networks were so confident in their prediction that the probability of the correct class effectively reached 1. This resulted in us being unable to calculate gradients. However, since the Carlini-Wagner loss function operates on the logits of the classifier, it enables the creation of adversarial examples.

2.2. Approach

Our approach aims at reducing the cost of sampling from the posterior while at the same time generating a strong attack. This is done by transforming the stochastic gradients in eq. (4) to deterministic ones by treating the BNN as a standard neural network where the posterior means serve as the weights value. We refer to this scheme of replacing the stochastic neural network with a deterministic one as `MEAN`. Note that this approach does not restrict the set of possible adversarial attacks. Under `MEAN` eq. (4) reduces to:

$$\boldsymbol{x}_{adv,\texttt{MEAN}} = \boldsymbol{x} + \epsilon \cdot sign\left(\nabla_{\boldsymbol{x}} L\left(\mathbb{E}(\boldsymbol{\theta}], \boldsymbol{x}, \boldsymbol{y}\right)\right) \quad . \tag{6}$$

Note the similarity of our approach to the rescaling of weights in dropout networks (Srivastava et al., 2014), where weights are scaled to have the same expectation under the droprate during testing as during training.

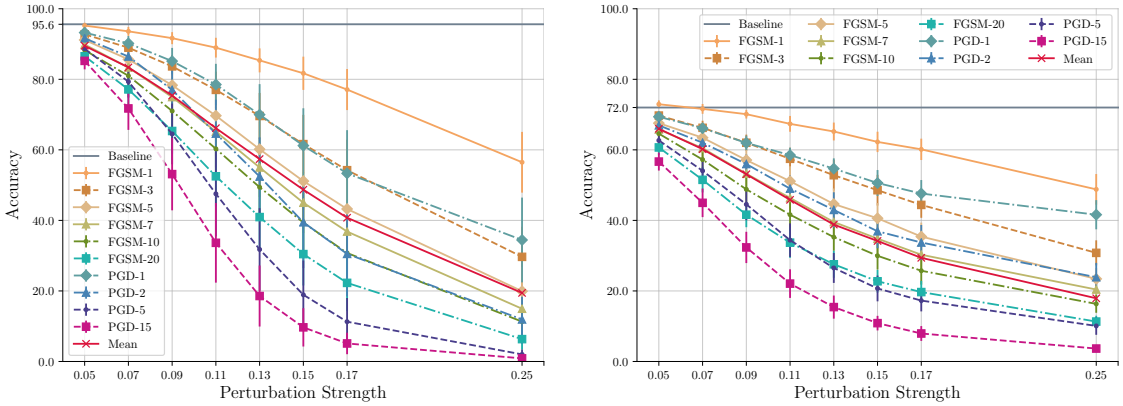
3. Experiments

In this section we evaluate the benefits of the `MEAN` scheme for creating AEs, in two settings: as an attack and during adversarial training. We first describe the general experimental setting in Section 3.1 and compare the effectiveness of adversarial attacks conducted with `MEAN` in Section 3.2. Finally, in Section 3.3 we leverage the `MEAN` approach in the context of adversarial training.

3.1. Experiment Setup

In our experiments we use the MNIST and Fashion MNIST data sets (LeCun et al., 1998; Xiao et al., 2017), both consisting of 60,000 training and 10,000 test images of size 28×28 labeled as one out of 10 classes. We trained two different CNN networks[1] using mean field

1. Details on the architecture and implementation can be found in Section A in the Appendix.



(*a*) MNIST (*b*) Fashion MNIST

Figure 1: **Accuracy of adversarial examples generated using different sample sizes from the posterior and the `MEAN` scheme.** We report the accuracy depending on the perturbation strength used (δ) for both MNIST and Fashion MNIST. For generating the AEs we used FGSM and PGD attack. We used FGSM exclusively for the `MEAN` scheme. For reference we include the original accuracy on the test set (Baseline). Best viewed in color.

variational inference and Adam (Kingma et al., 2015) as the optimizer. All our experiments are conducted on a Nvidia Quadro RTX 5000 graphic card.

AEs are generated using the FGSM and PGD (with 10 iteration steps) methods with varying perturbation strengths. Note that during the evaluation we refer to the FGSM attack computed over 10 samples drawn from the posterior as FGSM-10 (15 samples—FGSM-15; and so forth). Further, we always approximate the Bayesian predictive integral by 50 samples from the posterior.

Due to the inherent stochasticity we repeat all experiments 20 times, reporting the calculated mean as well as standard deviation in form of error bars for our results.

3.2. Generating Adversarial Examples

In this section we want to evaluate whether AEs generated by the `MEAN` scheme are comparable to examples generated by sampling from the posterior. Additionally, we will also evaluate the computational cost associated with the different approaches.

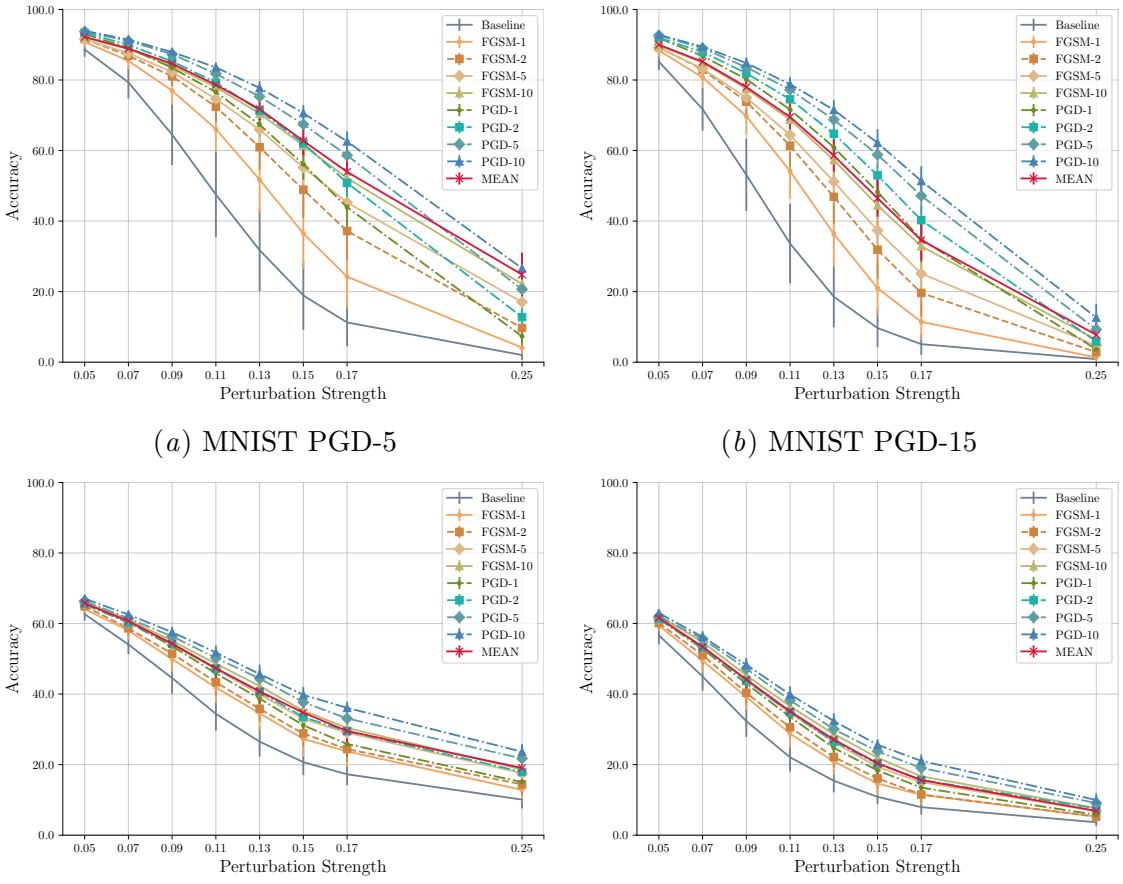
We generate 1,000 AE for $\epsilon \in \{0.05, 0.07, 0.09, 0.11, 0.13, 0.17, 0.25\}$ and depict the results in Figure 1. When compared to the FGSM attack, `MEAN` is competitive up to sampling five (MNIST)[2] or seven (Fashion MNIST) times from the posterior. We independently measured how long it takes to convert a batch of images to AEs, by converting 10,000 batches and computing the average time. The full results can be found in Table 2 in Appendix C. For MNIST, `MEAN` takes about 88ms to convert a single batch while FGSM-5 takes about 366ms, an increase in 315.91 %. The results are mirrored for Fashion MNIST, where FGSM-7 requires around 554 % more time; proving that `MEAN` can be used to approximate computationally more expensive attacks.

2. Results on the ratio of coincident gradient signs underlining this result can be found in the Appendix B.

When compared to the PGD attack, `MEAN` is stronger than PGD-1. It is on par with PGD-2 on Fashion MNIST, but otherwise outperformed. While surprising at first, we hypothesize that PGD, through iterative sampling, approximates the stochastic gradient quite well. Thus, even PGD-1 with 10 iteration steps can therefore be thought of as sampling 10 times from the posterior.

Note, we exclusively used FGSM to generate the AEs with the `MEAN` scheme. During our experiments we discovered that using PGD in conjunction with the deterministic network created by the `MEAN` scheme would overfit to this specific network, resulting in worse overall performance.

3.3. Adversarial Training



(*a*) MNIST PGD-5 (*b*) MNIST PGD-15

(*c*) Fashion MNIST PGD-5 (*d*) Fashion MNIST PGD-15

Figure 2: **Robustness comparison of differently adversarial trained models against PGD attacks.** All adversarial trained networks were attacked with the PGD attack for $\{5, 15\}$ draws from the posterior. A higher accuracy stands for a higher robustness against adversarial attacks. We include the accuracy of a normal trained network for reference (Baseline). Best viewed in color.

Table 1: **Training time for different adversarial training techniques.** We report the time it took to train a model in seconds and the increase relative to the `MEAN` in brackets.

Dataset	`MEAN`	FGSM-10		PGD-2		PGD-5		PGD-10	
MNIST	134s	208s	(+55 %)	304s	(+126 %)	592s	(+341 %)	1067s	(+696 %)
Fashion MNIST	336s	624s	(+85 %)	958s	(+185 %)	1871s	(+456 %)	3549s	(+956 %)

As demonstrated in Section 3.2 the `MEAN` attack leads to significantly stronger adversarial examples than computational equivalent attacks. While the resource overhead of drawing multiple samples might be condoned by an attacker, this practice would be infeasible if a defending party wants to compare multiple model architectures and hyperparameter combinations. Even though prior work suggest that using the strongest attack possible results in more robust models (Madry et al., 2017), sampling multiple times can quickly become infeasible. Additionally, robustness often requires a trade-off in regards to accuracy (Tsipras et al., 2019), leading to evaluating even more combinations in practice. In this scenario `MEAN` can serve as a good approximation of stronger training techniques. It can be used in the early stages of development to gauge the resistance to adversary examples, while the final model can be trained with the strongest attack possible.

We train multiple networks using different techniques to get a thorough understanding of how well we can utilize `MEAN` for training adversarial robust networks. We calculate adversarial examples on-the-fly during the training process, mixing "normal" and adversarial samples, i.e., with probability 0.3 we convert the current training batch to a batch of adversarial examples. The results can be found in Figure 2.

We plot the accuracy of different training schemes when attacked with PGD in different sample configurations ($\{5, 15\}$). Additionally, we provide the time it took to train the strongest models in Table 1 (a full table can be found in Appendix D). We can observe that `MEAN` can reliably approximate more costly techniques. It is only consistently outperformed by PGD-5 and PGD-10. Which require up to 956 % of the resources. Note that we only use adversarial examples in 30 % of the training, if we would supply the adversarial examples more often the discrepancy would be even higher.

4. Conclusion

Bayesian neural networks incorporate an inherent gradient masking effect during the calculation of adversarial examples due to their stochastic nature. Circumventing this masking comes at the cost of drawing multiple samples from the posterior for calculating the gradient. In this paper we proposed a simple and efficient solution to avoid the computational burden for BNNs, called `MEAN`. In this scheme, we transfer the BNN into a deterministic network, of equal architecture, by using the posterior means as weight values. In our experiments we found that using the `MEAN` scheme results in attacks as efficient as those produced by taking multiple samples from the gradient. Further, we found that `MEAN` has a high impact during adversarial training where it is only outperformed by PGD-5 and PGD-10, which require up to 956 % of the resources. Our findings suggest, that `MEAN` serves as a simple and cheap baseline for calculating adversarial examples. Therefore, `MEAN` is especially relevant during hyperparamerter tuning, extensive model comparison or costly network evaluations.

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Appendix

For the sake of completeness we extend this work by describing the implementation and adding further details on computational time for the results depict in the main part.

A. Network Architecture

The networks are constructed using pyro 1.3 and are based on the work of Carbone et al. (2020). For training the models we used mean-field variational inference with standard Gaussian distributions as priors. We used ADAM (Kingma and Ba, 2015) as our optimizer with an initial learning rate of 0.01 (MNIST) and 0.001(Fashion MNIST). Further, we trained for 5 (MNIST) and 10 (Fashion MNIST) epochs with a batch size of 128 for both data sets. As our activation function we used the Leaky ReLu function (Xu et al., 2015) with a negative slope of 0.01 throughout the network.

MNIST	**Fashion MNIST**
Conv 5x5 (32)	Conv 5x5 (32)
Max-Pooling (2)	Max-Pooling (2)
Conv 5x5 (512)	Conv 5x5 (1024)
Max-Pooling (1)	Max-Pooling (1)
Dense (10)	Dense (10)

The network architecture for our simple CNN. We report in brackets: For convolution layers the amounts of feature maps learned, for dense layers the amount of hidden units, and, for pooling layers the amount of stride. We use exclusively max-pooling with a kernel size of two.

B. Analyzing the Gradient Direction

In this section we want to investigate how accurate the approximation of `MEAN` is in comparison with sampling from the posterior. To this end, for a given image, we first obtain a ground truth gradient by sampling 100 times from the posterior. We then calculate how often the gradient entriespoint in the same direction as gradient entries obtained by using `MEAN` and sampling $\{1, 5, 10, 20\}$-times from the posterior. We investigate the model trained on MNIST and report the result for 1,000 test images.

We summarize our findings in Figure 3. For `MEAN` around 70% of the gradient entries have the same sign as the baseline. This outperforms gradients based on $\{1, 5, 10\}$ samples which is in line with the results from Section 3. The outliers in the plot are caused by examples where the gradient entries were either zero everywhere for the ground truth and/or the objects of interest. We hypothesize that these observation are caused by model structure in coherence to the findings of Carbone et al. (2020).

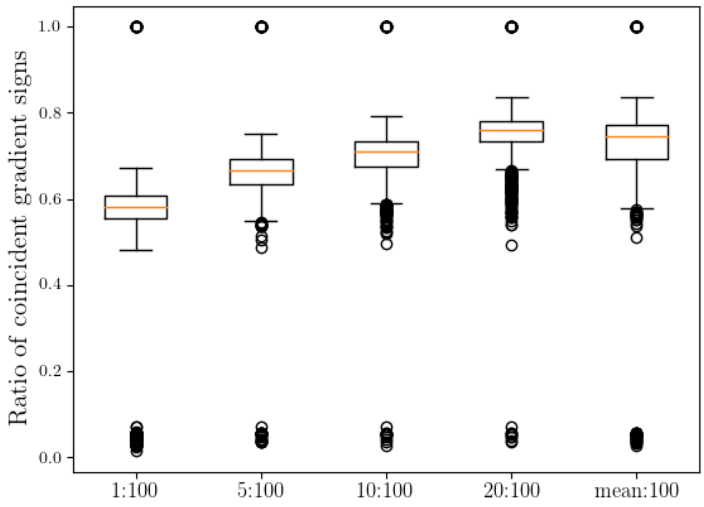

Figure 3: **Comparing the agreement of the direction of the gradient entries.** The boxplots show the results for 1,000 test images. For each image the signs of the gradient entries calculated based on $\{1, 5, 10, 20,$ `MEAN`$\}$ are compared to the signs of a gradient obtained from 100 samples from the posterior. We report the ratio of agreement.

C. Time Required to Create Adversarial Examples

In this section we report the time it took for converting AEs with different techniques. We convert a batch of images 10,000 times and average over all runs.

Table 2: **Time for converting AEs.** We report the time it takes to convert a single batch with the given technique, averaged over 10,000 samples. Additionally, we report the increase relative to the `MEAN` scheme in brackets.

Technique	MNIST		Fashion MNIST	
`MEAN`	88 ms		107 ms	
FGSM-1	122 ms	(+38.63 %)	119 ms	(+11.21 %)
FGSM-3	295 ms	(+235.22 %)	307 ms	(+186.92 %)
FGSM-5	366 ms	(+315.91 %)	504 ms	(+371.03 %)
FGSM-7	499 ms	(+467.05 %)	700 ms	(+554.21 %)
FGSM-10	614 ms	(+597.72 %)	996 ms	(+830.84 %)
FGSM-20	1,128 ms	(+1,181.81 %)	1,981 ms	(+1,751.40 %)
PGD-1	699 ms	(+ 694.32 %)	1,068 ms	(+898.13 %)
PGD-2	1,521 ms	(+ 1,628.41 %)	2,100 ms	(+1,862.62 %)
PGD-5	3,184 ms	(+ 3,518.18 %)	5,016 ms	(+4,587.85 %)
PGD-15	8,507 ms	(+ 9,567.05 %)	14,771 ms	(+13,704.67 %)

D. Training Time for Adversarial Training

In this section we report the time it took to adversarially train a network given the technique used to create AEs.

Table 3: **Training time for different adversarial training techniques.** We report the time it took to train a model in seconds and the increase relative to the `MEAN` in brackets.

Technique	MNIST		Fashion MNIST	
`MEAN`	134 s		336 s	
FGSM-1	133 s	(-0.87 %)	341 s	(+1.48 %)
FGSM-2	144 s	(+7.46 %)	372 s	(+10.71 %)
FGSM-5	169 s	(+26.12 %)	468 s	(+39.29 %)
FGSM-10	208 s	(+55.23 %)	642 s	(+91.07 %)
PGD-1	220 s	(+64.18 %)	650 s	(+93.45 %)
PGD-2	304 s	(+126.87 %)	958 s	(+185.12 %)
PGD-5	592 s	(+341.79 %)	1,871 s	(+456.85 %)
PGD-10	1,067 s	(+696.27 %)	3,549 s	(+956.25 %)